USN


10MAT31

Third Semester B.E. Degree Examination, June/July 2015
Engineering Mathematics - III
Time: 3 hrs.

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Expand $f(x)=x \sin x$ as a Fourier series in the interval $(-\pi, \pi)$, Hence deduce the following:
i) $\frac{\pi}{2}=1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}$
ii) $\frac{\pi-2}{4}=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-+\ldots$
(07 Marks)
b. Find the half-range Fourier cosine series for the function
$f(x)=\left\{\begin{array}{l}k x, \quad 0 \leq x \leq \ell / 2 \\ k(\ell-x), \ell / 2<x \leq \ell\end{array}\right.$
Where k is a non-integer positive constant.
(06 Marks)
c. Find the constant term and the first two harmonics in the Fourier series for $f(x)$ given by the following table.

| $\mathrm{x}:$ | 0 | $\pi / 3$ | $2 \pi / 3$ | $\pi$ | $4 \pi / 3$ | $5 \pi / 3$ | $2 \pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~F}(\mathrm{x}):$ | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

(07 Marks)
2 a. Find the Fourier transform of the function $f(x)=x e^{-a|x|}$
(07 Marks)
b. Find the Fourier sine transforms of the

Functions $f(x)=\left\{\begin{array}{cc}\sin x, & 0<x<a \\ 0, & x \geq a\end{array}\right.$
(06 Marks)
c. Find the inverse Fourier sine Transform of
$F_{x}(\alpha)=\frac{1}{\alpha} e^{-a \alpha} \quad a>0$.
(07 Marks)

3 a. Find various possible solution of one dimensional wave equation $\frac{\partial^{2} u}{\partial t^{2}}=C^{2} \frac{\partial^{2} u}{\partial x^{2}}$ by separable variable method.
(07 Marks)
b. Obtain solution of heat equation $\frac{\partial u}{\partial t}=C^{2} \frac{\partial^{2} u}{\partial t^{2}}$ subject to condition $u(0, t)=0, u(\ell, t)=0$, $u(x, 0)=f(x)$.
(06 Marks)
c. Solve Laplace equation $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ subject to condition $u(0, y)=u(\ell, y)=0, u(x, 0)=0$, $u(x, a)=\sin \left(\frac{\pi \mathrm{x}}{\ell}\right)$.
(07 Marks)
a. The pressure P and volume V of a gas are related by the equation $\mathrm{PV}^{\mathrm{r}}=\mathrm{K}$, where r and K are constants. Fit this equation to the following set of observations (in appropriate units)

| $\mathrm{P}:$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~V}:$ | 1.62 | 1.00 | 0.75 | 0.62 | 0.52 | 0.46 |

(07 Marks)
b. Solve the following LPP by using the Graphical method:

Maximize : $Z=3 x_{1}+4 x_{2}$
Under the constraints $4 \mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 80$

$$
\begin{aligned}
& 2 \mathrm{x}_{1}+5 \mathrm{x}_{2} \leq 180 \\
& \mathrm{x}_{1}, \mathrm{x}_{2} \geq 0
\end{aligned}
$$

(06 Marks)
c. Solve the following using simplex method

Maximize : $Z=2 x+4 y$, subject to the
Constraint : $3 \mathrm{x}+\mathrm{y} \leq 2 \mathrm{z}, \quad 2 \mathrm{x}+3 \mathrm{y} \leq 24, \quad \mathrm{x} \geq 0, \quad \mathrm{y} \geq 0$.
(07 Marks)

## PART - B

5 a. Using the Regular - Falsi method, find a real root (correct to three decimal places) of the equation $\cos x=3 x-1$ that lies between 0.5 and 1 (Here, $x$ is in radians).
(07 Marks)
b. By relaxation method

Solve : $-x+6 y+27 z=85,54 x+y+z=110,2 x+15 y+6 z=72$.
(06 Marks)
c. Using the power method, find the largest eigen value and corresponding eigen vectors of the matrix $A=\left[\begin{array}{ccc}6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3\end{array}\right]$
taking $[1,1,1]^{\mathrm{T}}$ as the initial eigen vectors. Perform 5 iterations.
(07 Marks)

6 a. From the data given in the following Table ; find the number of students who obtained
(i) Less than 45 marks $\quad$ ii) between 40 and 45 marks.

| Marks | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 31 | 42 | 51 | 35 | 31 |

(07 Marks)
b. Using the Lagrange's formula, find the interpolating polynomial that approximates to the function described by the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 3 | 6 | 11 | 18 | 27 |

Hence find $f(0.5)$ and $f(3.1)$.
(06 Marks)
c. Evaluate $\int_{0}^{1} \frac{x}{1+x^{2}} d x$ by using Simpson's $(3 / 8)^{\text {th }}$ Rule, dividing the interval into 3 equal parts. Hence find an approximate value of $\log \sqrt{2}$.
(07 Marks)

7 a. Solve the one - dimensional wave equation $\frac{\partial^{2} u}{\partial x^{2}}=\frac{\partial^{2} u}{\partial t^{2}}$
Subject to the boundary conditions $u(0, t)=0, u(1, t)=0, t \geq 0$ and the initial conditions $\mathrm{u}(\mathrm{x}, 0)=\sin \pi \mathrm{x}, \frac{\partial \mathrm{u}}{\partial \mathrm{t}}(\mathrm{x}, 0)=0,0<\mathrm{x}<1$.
(07 Marks)
b. Consider the heat equation $2 \frac{\partial^{2} \mathrm{u}}{\partial \mathrm{x}^{2}}=\frac{\partial \mathrm{u}}{\partial \mathrm{t}}$ under the following conditions:
i) $\mathrm{u}(0, \mathrm{t})=\mathrm{u}(4, \mathrm{t})=0, \mathrm{t} \geq 0$
ii) $u(x, 0)=x(4-x), 0<x<4$.

Employ the Bendre - Schmidt method with $\mathrm{h}=1$ to find the solution of the equation for $0<\mathrm{t} \leq 1$.
(06 Marks)
c. Solve the two - dimensional Laplace equation $\frac{\partial^{2} u}{\partial x^{2}}=\frac{\partial^{2} u}{\partial y^{2}}=0$ at the interior pivotal points of the square region shown in the following figure. The values of $u$ at the pivotal points on the boundary are also shown in the figure.
(07 Marks)


Fig. Q7 (c)

8 a. State and prove the recurrence relation of Z - Transformation hence find $\mathrm{Z}_{\mathrm{T}}\left(\mathrm{n}^{\mathrm{p}}\right)$ and
$Z_{T}\left[\cosh \left(\frac{n \pi}{2}+\theta\right)\right]$.
(07 Marks)
b. Find $Z_{T}^{-1}\left[\frac{z^{3}-20 z}{(z-2)^{3}(z-4)}\right]$
(06 Marks)
c. Solve the difference equation
$y_{n+2}-2 y_{n+1}-3 y_{n}=3^{n}+2 n$
Given $\mathrm{y}_{0}=\mathrm{y}_{1}=0$.
(07 Marks)


Third Semester B.E. Degree Examination, June/July 2015 Analog Electronic Circuits

Time: 3 hrs.
Max. Marks:100

## Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART - A

1 a. State and explain the various resistance levels of the semiconductor diode.
(06 Marks)
b. Explain the working of a full wave centre tapped rectifier. Also determine ripple factor, efficiency and voltage regulation.
( $\mathbf{1 0}$ Marks)
c. Design a suitable circuit represented by the box shown below, which has the input and output waveforms as indicated.
(04 Marks)



Fig. Q1(c)
2 a. Name different biasing methods of transistor. With circuit diagram analyze the fixed bias circuit, with effect of variation in $I_{B}, R_{c}$ and $V_{C C}$ on $Q$. point of the load line.
(10 Marks)
b. Explain the circuit of a transistor switch being used as an inverter.
(04 Marks)
c. In a voltage divider bias circuit of BJT. $\mathrm{V}_{\mathrm{CC}}=20 \mathrm{~V}, \mathrm{R}_{\mathrm{C}}=10 \mathrm{k} \Omega, \mathrm{RE}=1.5 \mathrm{k} \Omega$, $\mathrm{R}_{1}=40 \mathrm{k} \Omega, \mathrm{R}_{2}=4 \mathrm{k} \Omega$. Assume silicon transistor with $\beta=150$. Find $\mathrm{I}_{\mathrm{C}}, \mathrm{V}_{\mathrm{CE}}$ and $\mathrm{I}_{\mathrm{C}(\text { sat })}$ using exact analysis.
(06 Marks)
3 a. Define h - parameters and hence derive h - parameters model of CE - BIT.
(06 Marks)
b. Explain with a neat circuit diagram of emitter follower configuration. Justify how voltage gain is nearly equal to one.
(06 Marks)
c. For the circuit shown below determine $\mathrm{V}_{\mathrm{CC}}$, if $\mathrm{A}_{\mathrm{V}}=-160$ and $\mathrm{r}_{0}=100 \mathrm{k} \Omega$. Take $\beta=100$.
(08 Marks)


Fig. Q3(c)

4 a. Draw the single stage RC coupled BJT amplifier and discuss the effect of (low frequency response) : i) Input capacitance $\mathrm{C}_{\mathrm{S}}$ ii) output capacitance $\mathrm{C}_{\mathrm{C}}$ and iii) Emitter by pass capacitance $\mathrm{C}_{\mathrm{e}}$ on frequency response.
(05 Marks)
b. Prove that miller effect of input capacitance $\mathrm{C}_{\mathrm{Mi}}=\left(1-\mathrm{A}_{\mathrm{v}}\right) \mathrm{Cf}$ and output capacitance $\mathrm{C}_{\mathrm{M} 0}=\left(1-\frac{1}{\mathrm{~A}_{\mathrm{V}}}\right) \mathrm{C}_{\mathrm{f}}$.
(10 Marks)
c. It is desired that the voltage gain of an RC - coupled amplifier at 60 Hz should not decrease by more than $10 \%$ from its mid bond value. Calculate :
i) the lower 3 dB frequency
ii) the required C if $\mathrm{R}=2000 \Omega$.
(05 Marks)

## PART - B

5 a. Derive expressions for $Z_{i}$ and $A_{i}$ for a Darlington emitter follower circuit.
(10 Marks)
b. Mention the types of feedback connections. Draw their block diagrams indicating input and output signal.
(06 Marks)
c. List the general characteristics of a negative feedback amplifier and write its advantages.
(04 Marks)
6 a. With a neat circuit diagram, explain the operation of a transformer coupled class A power amplifier.
(07 Marks)
b. Explain the operation of a class B push-pull amplifier and derive its conversion efficiency.
(08 Marks)
c. The following distortion reading are available for a power amplifier : $\mathrm{D}_{2}=0.2, \mathrm{D}_{3}=0.02, \mathrm{D}_{4}=0.06$, with $\mathrm{I}_{1}=3.3 \mathrm{~A}$ and $\mathrm{R}_{\mathrm{C}}=4 \Omega$. Calculate : i) the THD ii) the fundamental power component iii) the total power.
(05 Marks)
7 a. Explain the working of Wien bridge oscillator.
(07 Marks)
b. With a neat circuit diagram, explain the operation of BJT Colpitts oscillator.
(06 Marks)
c. A crystal has the following parameter $\mathrm{L}=0.334 \mathrm{H}, \mathrm{C}_{\mathrm{M}}=1 \mathrm{pF}, \mathrm{C}=0.065$ and $\mathrm{R}=5.5 \mathrm{k} \Omega$. Calculate the series resonant frequency, parallel resonant frequency and find Q of the crystal.
(07 Marks)
8 a. Draw the JFET common drain configuration (source - follower) circuit. Derive $\mathrm{Zi}, \mathrm{Z}_{0}$ and $\mathrm{A}_{\mathrm{v}}$ using small signal model. Write its characteristics.
(10 Marks)
b. Compare JFET and MOSFET.
(03 Marks)
c. For the JFET common drain configuration shown below. Given $I_{d s s}=10 \mathrm{~mA}, \mathrm{~V}_{\mathrm{P}}=-5 \mathrm{~V}$, $r_{d}=40 \mathrm{k} \Omega, V_{G S O}=-2.85 \mathrm{~V}$ i) Calculate $Z_{i}$ and $Z_{0}$ ii) Calculate $A_{v}$ iii) find $V_{0}$ if $V_{i}=20 \mathrm{mV}$ ( $\mathrm{p}-\mathrm{p}$ ).
(07 Marks)


Fig. Q8(c)

$$
2 \text { of } 2
$$



10ES33

Third Semester B.E. Degree Examination, June/July 2015 Logic Design
Time: 3 hrs.
Max. Marks:100

## Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

## PART - A

1 a. Express the following Boolean function in canonical min term form :

$$
\mathrm{F}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\overline{\mathrm{A}} \overline{\mathrm{~B}}+\mathrm{C} .
$$

(04 Marks)
b. Express the following Boolean function in canonical max term form :
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\overline{\mathrm{A}} \mathrm{B}+\mathrm{C} \overline{\mathrm{D}}$.
(08 Marks)
c. Simplify the following Boolean function using four variable ' k ' map. Realize the simplified expression using NAND gates.
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(1,5,6,7,11,12,13,15)$.
(08 Marks)

2 a. Simplify the following Boolean function using Quine - Moclusky's minimization technique. $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(6,7,9,10,13)+\mathrm{d}(1,4,5,11,15)$.
(10 Marks)
b. Consider the following Boolean equation
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(1,3,7,11,15)+\sum \mathrm{d}(0,2,5)$.
Simplify the function F using a 3 variable MEV k - map. Assign the variable D to be the MEV.
(10 Marks)

3 a. Implement the Boolean functions :
$F_{1}(x, y, z)=X \bar{Y}+Y Z$
$\mathrm{F}_{2}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\pi \mathrm{m}(0,3,5)$
Using a 3-8 line decoder IC 74138 with active low outputs.
(08 Marks)
b. Interface a 10 key keypad to a digital system using a IC 74147 which is a 10 line to BCD priority encoder. Draw the logic diagram and explain the operation with the truth table.
(12 Marks)

4 a. Implement the Boolean function :
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(0,1,2,4,5,7,8,9)$
Using a 8 to 1 multiplexer. Draw the logic diagram and explain the operation. Additional gates can be used if required.
(08 Marks)
b. Explain the operation of a full subtractor with the help of a truth table and Boolean expressions for the outputs. Implement the full subtractor using two numbers of
i) 4 to 1 multiplexers
ii) 2 to 1 multiplexers.

Additional gates if required can be used.
c. Design a one bit binary comparator.


## PART - B

5 a. Explain the operation of a gated SR latch with a logic diagram and a truth table. (06 Marks)
b. Explain the operation of a positive edge trigged 'D' flip-flop with the help of a logic diagram and truth table. Also draw the relevant waveforms.
(04 Marks)
c. Draw the output waveforms $Q_{M}$ and $Q_{S}$ the outputs of the master and the slave respectively, if the inputs to a master slave JK flip-flop one as indicated below.
( 10 Marks)


Fig. Q5(c)
6 a. Design a 4 bit binary ripple up counter using negative edge triggered JK flip-flops. Draw the timing diagram with respect to the input cock pulses. Explain the operation.
( 10 Marks)
b. Design a synchronous counter using clocked JK flip-flop for the counting sequence shown below:

| $\mathrm{Q}_{2}$ | $\mathrm{Q}_{1}$ | $\mathrm{Q}_{0}$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

(10 Marks)
7 a. Explain mealy and Moore models of a clocked synchronous sequential circuit. (08 Marks)
b. Design a synchronous circuit using positive edge triggered JK flip-flops to generate the following sequence :
$0-1-2-0$ is input $x=0$ and
$0-2-1-0$ is input $x=1$
Provide an output which goes high to indicate the non - zero states in the $0-1-2-0$ sequence.
( 12 Marks)
8 Construct the excitation table, transition table, state table and state diagram for the sequential circuit shown in Fig. Q8.
(20 Marks)


Fig. Q8

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## Third Semester B.E. Degree Examination, June/July 2015

Network Analysis
Time: 3 hrs.

Max. Marks: 100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## $\underline{\text { PART - A }}$

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

1 a. Derive expression for
i) Star to delta transformation ii) Delta to star transformation
(10 Marks)
b. For the Network shown find the node voltages Vd and Vc Fig. Q No. 1 (b). ( $\mathbf{1 0}$ Marks)

Fig.Q1(b)


2 a. Define the following with examples
i) Oriented graph ii) Tree iii) Fundamental cut set iv) Fundamental tie set (08 Marks)
b. For the network, Shown Fig. Q No. 2 (b) write the tie set schedule, tie set matrix and obtain equilibrium equation in matrix form using KVL. Calculate branch currents and branch voltage. Follow the same orientation and branch numbers use 4,5 and 6 as tree branches.
(12 Marks)

Fig.Q2(b)


3 a. State and prove Reciprocity theorem.
(07 Marks)
b. Find the output voltage Eo of the Network shown Using Millman's theorem. Fig. Q No. 3(b)

(06 Marks)
c. Using superposition theorem, find the current IX the network shown in Fig. Q No.3(c)

Fig.Q3(c)

(07 Marks)

4 a. State Norton's theorem. Show that Thevenin's equivalent circuit is the dual of Norton's equalent circuit.
(06 Marks)
b. Obtain the current $\mathrm{I}_{\mathrm{x}}$ by using Thevenin's theorem for the network shown in Fig Q No.4(b)

(08 Marks)
c. State maximum power transfer theorem. Prove that $\mathrm{Z}_{\mathrm{L}}=\mathrm{Z}_{\mathrm{o}}$ * for Ac circuits.

## PART - B

5 a. Show that $\mathrm{f}_{0}=\sqrt{\mathrm{f}_{1} \mathrm{f}_{2}}$ fro series Resonance circuit.
(06 Marks)
b. A voltage of 100 sin $w t$ is applied to an RLC series circuit at resonant frequency. The voltage across a capacitor was found to be 400 V . The bandwidth is 75 Hz . The impedence at resonance is $100 \Omega$. Find the resonant frequency and constants of the circuit.
(06 Marks)
c. Derive an expression for the resonant frequency of a resonant frequency of a resonant circuit consisting of $R_{L} L$ in parallel with $R_{c} C$. Draw the frequency response curve of the above circuit.
(08 Marks)

6 a. In the circuit shown, switch $K$ is changed from 1 to 2 at $t=0$, steady state having been attained in position 1 . Find the values of $i, \frac{d i}{d t}$ and $\frac{d^{2} i}{\mathrm{dt}^{2}}$ at $t=0^{+}$.
(10 Marks)

Fig.Q6(a)

b. In the circuit shown, switch K is kept open for very long time, on closing K , after 10 ms , $\mathrm{Vc}=80 \mathrm{~V}$. Then the switch K is kept closed for a long time. When the switch is opened again, $\mathrm{Vc}=90 \mathrm{~V}$ after half second, calculate values of R and C . Fig. Q No.6 (b)

Fig.Q6(b)

(10 Marks)
7 a. State and prove i) Initial value theorem ii) Final value theorem as applied to Laplace transform. What are the limitations of each theorem.
( 10 Marks)
b. In the circuit shown, in Fig.Q No. 7 (b) switch is initially closed. After steady the switch is opened, Determine the nodal voltages $\mathrm{V}_{\mathrm{a}}(\mathrm{t})$ and $\mathrm{V}_{\mathrm{b}}(\mathrm{t})$ using Laplace transform method.

(10 Marks)
8 a. Define z-parameters. Express z-parameters in terms of y - parameters.
(10 Marks)
b. Find $y$ parameters and $z$ parameters for the circuit shown.


Fig.Q8(b)
10 Marks)


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## Third Semester B.E. Degree Examination, June/July 2015

Electronics Instrumentation
Time: 3 hrs .
Max. Marks: 100

## Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART - A

1 a. Define the following with example:
i) Accuracy
ii) Precision
iii) Resolution
iv) Significant figures.
(08 Marks)
b. With a schematic, explain a true rms voltmeter.
(06 Marks)
c. Calculate the value of the multiplier resistor for a 50 V rms ac range on the voltmeter as shown in Fig.Q.1(c).
(06 Marks)


Fig.Q.1(c)
2 a. Give the working principle of following:
i) V - F type DVM
ii) Successive approximation.
(12 Marks)
b. Explain the principle, construction and working of a digital frequency meter. (08 Marks)

3 a. Draw the basic block diagram of an oscilloscope and explain the functions of each block and mention the advantages of negative HV supply.
(10 Marks)
b. Explain dual trace oscilloscope with a neat block diagram.
(10 Marks)
4 a. Explain the need for a delayed time base oscilloscope. Draw the block diagram of a delayed time base, and explain how it operates.
(10 Marks)
b. With block diagram, explain the operation of an analog storage oscilloscope.
(10 Marks)

## PART - B

5 a. Explain the operation of conventional standard signal generator with the help of block diagram.
b. Write a brief note on function generator.
c. Explain the operation of a sweep frequency generator with the help of a suitable block diagram. Mention its applications.
(10 Marks)

6 a. Explain the operation of the Maxwell's bridge, with a neat circuit diagram. Derive an expression for unknown values of resistance and inductance. What are the limitations of Maxwell's bridge?
(10 Marks)
b. Explain the operation of the capacitance comparison bridge, with a neat circuit diagram and derive the necessary equations.
(06 Marks)
c. A capacitance comparison bridge is used to measure a capacitive impedance at a frequency of 2 kHz . The bridge constants at balance are $\mathrm{C}_{3}=100 \mu \mathrm{~F}, \mathrm{R}_{1}=10 \mathrm{~K} \Omega, \mathrm{R}_{2}=50 \mathrm{~K} \Omega$, $R_{3}=100 \mathrm{~K} \Omega$. Find the equivalent series circuit of the unknown impedance.
(04 Marks)
7 a. List at least five advantages of electrical transducer.
(05 Marks)
b. Explain the method of measuring displacement using LVDT with a suitable diagram. State the advantages and disadvantages of LVDT.
(10 Marks)
c. Write a note on differential output transducers.

8 a. Write a note on photo transistor.
b. List at least five classifications of digital displays.
c. Explain the operation of the measurement of power by means of bolometer bridge, with the suitable circuit.
(10 Marks)
$\square$

## Third Semester B.E. Degree Examination, June/July 2015 Advanced Mathematics - I

Time: 3 hrs .
Max. Marks; 100

## Note: Answer any FIVE full questions.

1 a. Express the complex number

$$
\frac{(5-3 i)(2+i)}{4+2 i} \text { in the form } x+i y
$$

(06 Marks)
b. Find the modulus and the amplitude of $1+\cos \theta+i \sin \theta$.
(07 Marks)
c. Find the cube roots of $1+\mathrm{i}$.
(07 Marks)
2 a. Find the $\mathrm{n}^{\text {th }}$ derivative of $\mathrm{e}^{\mathrm{ax}} \cos (\mathrm{bx}+\mathrm{c})$.
(06 Marks)
b. Find the $\mathrm{n}^{\text {th }}$ derivative of $\frac{\mathrm{x}}{(\mathrm{x}+1)(2 \mathrm{x}+3)}$.
(07 Marks)
c. If $x=\tan (\log y)$ prove that $\left(1+x^{2}\right) y_{n+1}+(2 n x-1) y_{n}+n(n-1) y_{n-1}=0$.
(07 Marks)
3 a. Find the angle of intersection of the curves $r^{n}=a^{n} \cos n \theta, r^{n}=b^{n} \sin n \theta$.
(06 Marks)
b. Find the Pedal equation of the curve $r=a(1-\cos \theta)$.
c. Using Maclcaurin's series expand $\log (1+x)$ upto the term containing $x^{4}$.
(07 Marks)
4 a. If $u=f(x+c t)+g(x-c t)$ show that $\frac{\partial^{2} u}{\partial t^{2}}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$.
(06 Marks)
b. If $u=f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x u_{x}+y u_{y}+z u_{z}=0$.
(07 Marks)
c. If $u=x+y, v=y+z, w=z+x$ find the value of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.
(07 Marks)

5 a. Obtain the reduction formula for $\int \cos ^{n} \mathrm{xdx}$ where n is a positive integer.
(06 Marks)
b. Evaluate $\int_{0}^{a} \frac{x^{4}}{\sqrt{a^{2}-x^{2}}} d x$.
(07 Marks)
c. Evaluate $\int_{0}^{a} \int_{0}^{x} \int_{0}^{x+y} e^{x+y+z}, d z d y d x$.
(07 Marks)

6 a. Define beta and gamma functions and prove that $\Gamma(n+1)=n \Gamma(n)$.
(06 Marks)
b. Show that $\int_{0}^{\pi / 2} \sqrt{\sin \theta} \mathrm{~d} \theta \times \int_{0}^{\pi / 2} \frac{1}{\sqrt{\sin \theta}} \mathrm{~d} \theta=\pi$.
(07 Marks)
c. Prove that $\beta(\mathrm{m}, \mathrm{n})=\frac{\Gamma(\mathrm{m}) \cdot \Gamma(\mathrm{n})}{\Gamma(\mathrm{m}+\mathrm{n})}$.
(07 Marks)

7 a. Solve : $\frac{d y}{d x}=\cos (x+y+1)$.
b. Solve : $\left(x^{2}-y^{2}\right) d x-x y d y=0$.
(07 Marks)
c. Solve : $\frac{d y}{d x}+y \cot x=4 x \operatorname{cosec} x$.

8 a. Solve : $\left(D^{3}-6 D^{2}+11 D-6\right) y=0$.
(06 Marks)
b. Solve: $\left(D^{2}+2 D+1\right)=x^{2}+e^{+x}$.
(07 Marks)
c. Solve: $\left(D^{2}+D+1\right) y=\sin 2 x$.


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